RESEARCH SUMMARY AND PLAN

Xuerong Yong

My research interests lie in discrete and discrete applied mathematics, and theoretical computer science. Most of my work is concentrated in the areas of algebraic graph theory, linear algebra, combinatorics, and analysis of algorithms. List of *publications* is included in my CV. Below is a simple summary which is of four parts.

1. Counting Structures in Graphs: The *structures*, e.g., spanning trees, Hamiltonian cycles, independent sets, acyclic orientations, cycle covers, k-colorings etc., are important quantities in graph theory and they have many different applications. It is interesting to derive formulas for counting these structures for certain kinds of graphs. Recently, generalizing the standard *transfer matrix technique*, for certain classes of graphs, we proved that the numbers of these structures satisfy linear recurrence relations. Here is a brief description on the spanning trees problem.

Counting the number of spanning trees is a problem of long-standing interest in mathematics and physics and, in applications, it is also important in the study of the reliability of a *network* in the presence of line faults. Although obtaining the exact number of spanning trees is usually difficult, for certain classes of graphs, deriving explicit or recurrence formulas has been proven to be possible. Here is a very simple example: Boesch et al. and Bedrosian conjectured independently that if C^2 is a square cycle of n nodes, then the number of spanning trees of C^2 is nF_n^2, where $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$, $F_1 = 1$ are the *Fibonacci numbers.* This conjecture was proven independently by D. Kleitman and B. Golden, F. Boesch and H. Prodinger, and myself and F. Zhang (where our proof is simplest). Since then we have developed *techniques* for deriving general formulas for the number of spanning trees in different classes of graphs and, in particular, we proved that the number of spanning trees of any circulant graph satisfies a linear recurrence relation. These formulas greatly reduce the amount of time needed to calculate the number of spanning trees. Most recently, we obtained efficient *techniques* for attacking their combinatorial properties: recurrence relations and asymptotic functions etc.

2. Special Matrices: In this area my work was focused on *nonnegative matrices (e.g.,* $(0,1)$*-matrices), M-matrices, H-matrices, Elliptic matrices, etc.*, with special attention paid to the spectral radius, the inversion, and the distribution of eigenvalues as well as their applications (e.g., the stability analysis of a dynamical system). My deepest result is the proof of a conjecture posed by R. Horn and C. Johnson in their book "Topics in Matrix Analysis, Cambridge, 1991." (*Independently*, the same conjecture was also posed by Fiedler and Markham in Fiedler's book "Special Matrices and Their Applications in Numerical Mathematics, Dordrecht & Praha, 1986." It states that: let A be an $n \times n$ nonsingular M-matrix, A^{-1} its inverse, and let $A \circ A^{-1}$ be the Hadamard product of A and A^{-1}. Then $q(A \circ A^{-1}) \geq \frac{2}{n}$, where $q(B)$ stands for the least eigenvalue (in modulus) of B. Before I proved its validity, an incorrect proof had previously appeared in the literature and $q(A \circ A^{-1}) \geq \frac{1}{n}$ was known and used. Most recently, we obtained new properties for hyper-tournament related matrices.

3. Constrained Codes: I am also interested in the area of Shannon theory of constrained codes. My work was focused on analyzing the *channel capacities* of two-dimensional constrained codes. The problems are closely related to certain kinds of *counting problems in combinatorics*, e.g., counting the sequences which satisfy certain restrictions, and are motivated by the problems in information theory arising from the codes for mass data storage/transmission systems. While the study of the Shannon capacity of one dimensional codes is well developed, the study of two dimensional codes is still in its infancy (although recently very active). In our work, we established new theoretical results on two-dimensional constrained codes and developed general techniques for bounding their capacities. Our main tools are *matrix theory, graph spectra and combinatorial analysis.* As examples, we considered and improved the analysis of the capacity of runlength limited $(1, \infty)$ two-dimensional constrained codes (previously considered by N. Calkin and H. Wilf), the channel capacity of read/write isolated memory (previously considered by M. Cohn), and the number of placements of non-attacking kings (a combinatorial problem due to D. Knuth previously considered by H. Wilf) etc.. We were also able to apply the techniques developed to attack other non information-theory related problems.

4. Spectra of Graphs: The topological structure of a graph can be analyzed by studying the distribution of eigenvalues of the graph; The spectral technique can be used to attack certain counting problems in combinatorics; In applied sciences, e.g., statistical physics, quantum chemistry, etc., the eigenvalues of graphs represent energy levels of molecules. I have been interested in this topic since 1996. My deepest result can be described as follows: In the set of graphs $\{G_m\}$ of order n and with negative third largest eigenvalue, there is only one graph that has no eigenvalue equal to -1, all others have the property that there is an index $k \leq \frac{n}{2}$ such that $\lambda_j = -1,\ j = k, k+1, \cdots, n-k+1$, where λ_k is the kth largest eigenvalue of G (in some cases the j can run up to $n-k+2$).

Short Term Research Plan: Research on algebraic graph theory, matrix theory, and combinatorics is very interesting and there has been much interesting work to do. I will continue working on the problems relevant to these areas. Also, combining the techniques from advanced matrix analysis, algebraic graph theory and combinatorics is found to be very efficient to attack many problems from discrete applied mathematics. Recently, there has been active work in applied sciences (e.g. the ice-type model, the dimer problem in statistical physics and chemistry) that utilizes these techniques. So, in a few coming years, I am also expanding my knowledge into their related areas. Precisely, I will concentrate much of my work on applied graph theory and combinatorics problems and on their applications in the areas of mathematical sciences and their related applied sciences.